UDC 541.124+532.5

# THE TWO-POINT BOUNDARY VALUE PROBLEM WITH A SMALL PARAMETER IN THE THEORY OF A REACTOR WITH A NONHOMOGENEOUS FLUIDIZED BED

PMM Vol. 40, № 4, 1976, pp. 641-647

B. M. MARKEEV
(Moscow)

(Received October 20, 1975)

A method based on the asymptotic joining of expansions is developed for solving the two-point boundary value problem for a system of equations with a single small parameter (high Péclet number) that occur in the theory of isothermal reactors with nonhomogeneous fluidized bed. External analytic solution in the form of a series containing three terms is derived for the case of small coefficient of exchange between bubbles and medium. An analytic solution of the problem is presented. It is uniformly valid over the whole segment and accurate to within quadratic terms containing the small parameter, which appear in the two-point boundary value problem.

Models of reactors with a nonhomogeneous fluidized bed (see, e.g. [1—4]) are based on the concept of the fluidized bed as a two-phase system consisting of a continuous phase moving in the reactor at the initial fluidization rate, and of a discrete phase which is the excess of the fluidizing agent which moves through the bed in the form of bubbles. The computation of such type of reactor in the case of a reaction of an arbitrary order is complicated even in the one-dimensional case and can only be carried out by numerical methods. Because of this, approximate methods of nonlinear mechanics are of particular interest, for instance, for the computation of the steady mode of reactor operation.

A method of solving the steady state equation for the isothermal reactor at high Péclet numbers was developed, based on the joining of asymptotic expansions. Later, a method similar to that of joining asymptotic expansions was proposed in [7] for solving equation of the isothermal reactor. However the extension of that method to systems of equations presents some difficulties [8—10]. Due to the nonlinearity of equations of the nonisothermal reactor it was not possible to obtain an external analytic solution in the zero and subsequent approximations.

**1.** The two-point boundary value problem occurring in the theory of reactors with a nonhomogeneous fluidized bed is formulated on several assumptions as follows:

$$\varepsilon \frac{d^2x}{d\xi^2} = \frac{dx}{d\xi} - Kx^n + S(x-y), \quad \frac{dy}{d\xi} = S\frac{u}{v}(x-y) \qquad (1.1)$$

$$0 \leqslant \xi \leqslant 1$$

$$x(+0) = 1 + \varepsilon \frac{d}{d\xi} x(+0), \quad \frac{dx(1)}{d\xi} = 0, \quad y(+0) = 0$$

where $\xi = z / L$ is a dimensionless coordinate; $x = x^* / x^*(-0)$ and $y = y^* / y^*(-0)$ define the dimensionless concentration of the reacting substance in the continuous phase and in the bubble, $x^*(-0)$ and $y^*(-0)$ is the concentration at the reactor inlet; $S = S^* / (Lu)$ is the dimensionless coefficient of exchange between bubble and continuous phase; $u$ is the velocity of the continuous phase and $v$ is the velocity of bubbles; $\varepsilon^{-1} = \mathrm{Pe} = Lu\,/\,D$ is the Péclet number and $D$ is the diffusion coefficient; $K = K^*x^n(-0)\,/\,(Lu)$ is the dimensionless rate of reaction, and $n$ is the order of the reaction taking place in the reactor.

Let us consider system (1.1) when $\varepsilon \ll 1$. We seek the solution by the method of successive perturbations

$$x(\xi) = \chi_x(\xi), \quad y(\xi) = \chi_y(\xi), \quad \chi_{x,\,y}(\xi) = \sum_{n=0}^{\infty} \varepsilon^n \chi_{x,\,y}^{(n)}(\xi) \tag{1.2}$$

Substituting (1.2) into (1.1) and setting $y\ (-0) = 0$, we obtain the following system of zero approximation equations:

$$\frac{d\chi_x^{(0)}}{d\xi} - K\chi_x^{(0)n} + S(\chi_x^{(0)} - \chi_y^{(0)}) = 0, \quad \frac{d\chi_y^{(0)}}{d\xi} - \frac{u}{v} S(\chi_x^{(0)} - \chi_y^{(0)}) = 0 \tag{1.3}$$

$$\chi_x^{(0)}(+0) = 1, \quad \chi_y^{(0)}(+0) = 0$$

We restrict the analysis to the case of small coefficients of exchange between bubbles and the continuous phase. We represent the solutions of the zero and subsequent approximations in terms of $\varepsilon$ by the series in the small number $S$

$$\chi_{x,\,y}^{(l)}(\xi) = \sum_{m=0}^{\infty} S^m \chi_{x,\,y(m)}^{(l)}(\xi) \tag{1.4}$$

**2.** Let us investigate the case of second order reaction $(n = 2)$ for the zero approximation system

$$d\chi_{x(0)}^{(0)}\,/\,d\xi - K\chi_{x(0)}^{(0)2} = 0, \quad dx_{y(0)}^{(0)}\,/\,d\xi = 0 \tag{2.1}$$

$$\chi_{x(0)}^{(0)}(+0) = 1, \quad \chi_{y(0)}^{(0)}(+0) = 0$$

whose solution is

$$\chi_{x0}^{(0)}(\xi) = [-K\xi + 1]^{-1}$$

Using the zero approximation system (2.1) we find that the system of first approximation equations in terms of small number $S$

$$d\chi_{x(1)}^{(0)}\,/\,d\xi - 2K\chi_{x(0)}^{(0)}\chi_{x(1)}^{(0)} + (\chi_{x(0)}^{(0)} - \chi_{y(0)}^{(0)}) = 0 \tag{2.2}$$

$$\frac{d\chi_{y(1)}^{(0)}}{d\xi} - \frac{u}{v}(\chi_{x(0)}^{(0)} - \chi_{y(0)}^{(0)}) = 0, \quad \chi_{x(1)}^{(0)}(+0) = \chi_{y(1)}^{(0)}(+0) = 0$$

can be represented in the form

$$\chi_{x(1)}^{(0)}(\xi) = \frac{1}{2K}(1 - \chi_{x(0)}^{(0)2}(\xi)), \quad \chi_{y(1)}^{(0)}(\xi) = \frac{u}{vK}\ln\chi_{x(0)}^{(0)}(\xi)$$

Similarly to (2.2) the system of second approximation equations in small $S$ is a system of linear nonhomogeneous equations whose solution can be represented in the form

$$\chi_{x(2)}^{(0)}(\xi) = \left(\frac{\chi_{x(0)}^{(0)}(\xi)}{K}\right)^2 \left[\frac{1}{12}\frac{1 - \chi_{x(0)}^{(0)3}(\xi)}{\chi_{x(0)}^{(0)3}(\xi)} + \frac{1}{4}(\chi_x^{(0)}(\xi) - 1) - \right.$$

$$\left.\frac{u}{9v}\left(\frac{3\ln\chi_{x(0)}^{(0)}(\xi) + 1}{\chi_{x(0)}^{(0)3}(\xi)} - 1\right)\right]$$

$$\chi_{x(2)}^{(0)}(\xi) = \frac{u}{vK^2}\left\{-\frac{1}{2}\frac{1 - \chi_{x(0)}^{(0)}(\xi)}{\chi_{x(0)}^{(0)}(\xi)} - \frac{1}{2}(\chi_{x(0)}^{(0)}(\xi) - 1) + \right.$$

$$\frac{u}{v}\left(\frac{\ln \chi_{x(0)}^{(0)}(\xi)-1}{\chi_{x(0)}^{(0)}(\xi)}-1\right)\Bigg\}$$

The solution of the system of nonhomogeneous linear equations for subsequent approximations can be always expressed at least in terms of quadratures.

Let us now consider the system of first approximation equations in $\varepsilon$. Substituting (1. 2) into (1. 1), for the first approximation we obtain

$$\frac{d\chi_x^{(1)}}{d\xi}-2K\chi_x^{(0)}\chi_x^{(1)}+S\,(\chi_x^{(1)}-\chi_y^{(1)})=-\frac{d^2\chi_x^{(0)}}{d\xi^2} \tag{2.3}$$
$$\frac{d\chi_y^{(1)}}{d\xi}-S\frac{u}{v}(\chi_x^{(1)}-\chi_y^{(1)})=0,\quad \chi_x^{(1)}(+0)=K-S,\quad \chi_y^{(1)}(+0)=0$$

The solution of this system in the case of small coefficients of surface exchange is sought, as previously, in the form (1. 4). The zero approximation system

$$\frac{d\chi_{x(0)}^{(1)}}{d\xi}-2K\chi_{x(0)}^{(0)}\chi_{x(0)}^{(1)}=\frac{d^2\chi_{x(0)}^{(0)}}{d\xi^2},\quad \frac{d\chi_{y(0)}^{(1)}}{d\xi}=0$$
$$\chi_{x(0)}^{(1)}(+0)=K,\quad \chi_{y(0)}^{(1)}(+0)=0$$

has the following solution:

$$\chi_{x(0)}^{(1)}(\xi)=K\chi_{x(0)}^{(0)2}(\xi)\,[1+\ln\chi_{x(0)}^{(0)}(\xi)],\quad \chi_{y(0)}^{(1)}(\xi)=0 \tag{2.4}$$

The solution of system (2. 1) was used for the derivation of (2. 4).

From (2. 3) we similarly obtain in the first approximation in terms of $S$ a system of equations which has the following solutions:

$$\chi_{x(1)}^{(1)}(\xi)=\chi_{x(0)}^{(0)2}(\xi)\,\{2\,[2\chi_{x(0)}^{(0)}(\xi)-1)-\chi_{x(0)}^{(0)}(\xi)\ln\chi_{x(0)}^{(0)}(\xi)]-1\}$$
$$\chi_{y(1)}^{(1)}(\xi)=\frac{u}{v}\{2\chi_{x(0)}^{(0)}(\xi)\ln\chi_{x(0)}^{(0)}(\xi)-(\chi_{x(0)}^{(0)}(\xi)-1)\}$$

Finally, by substituting into the system of second approximation equations

$$\frac{dx_x^{(2)}}{d\xi}-2K\chi_x^{(0)}\chi_x^{(0)}+S\,(\chi_x^{(2)}-\chi_y^{(2)})=\frac{d^2\chi_x^{(1)}}{d\xi^2}+K\chi_x^{(1)2} \tag{2.5}$$
$$\frac{d\chi_y^{(2)}}{d\xi}-\frac{u}{v}S\,(\chi_x^{(2)}-\chi_y^{(2)})=0$$
$$\chi_x^{(2)}(+0)=2\,(K-S)(2K-S)+\frac{u}{v}S^2,\quad \chi_y^{(2)}(+0)=0$$

the solution in the form of series in $S$, we obtain for the zero approximation in terms of $S$ the system of equations

$$\frac{d\chi_{x(0)}^{(2)}}{d\xi}-2K\chi_{x(0)}^{(0)}\chi_{x(0)}^{(2)}=\frac{d^2\chi_{x(0)}^{(1)}}{d\xi^2}+K\chi_{x(0)}^{(1)2}$$
$$d\chi_{y(0)}^{(2)}/d\xi=0,\quad \chi_{x(0)}^{(2)}(+0)=4K^2,\quad \chi_{y(0)}^{(2)}(+0)=0$$

whose solutions are

$$\chi_{x(0)}^{(2)}(\xi) = (K\chi_{x(0)}^{(0)}(\xi))^2 \{9(\chi_{x(0)}^{(0)}(\xi) - 1) +$$
$$[(\chi_{x(0)}^{(0)}(\xi) \ln \chi_{x(0)}^{(0)}(\xi))(2 + \ln \chi_{x(0)}^{(0)}(\xi)) + 1]\}, \quad \chi_{y(0)}^{(2)}(\xi) = 0$$

Solutions of system (2.5) in subsequent approximations in $S$ are similarly obtained.

**3.** For obtaining a solution of system (1.1) that satisfies boundary conditions at the end of the segment we introduce in the neighborhood of point $\xi = 1$ a "boundary layer" [7]. After the substitution of the variable $\eta = (1 - \xi) / \varepsilon$ the input system of Eqs. (1.1) assumes the form

$$\frac{d^2\psi_x}{d\eta^2} + \frac{d\psi_x}{d\eta} = \varepsilon\,[-K\psi_x{}^n + S(\psi_x - \psi_y)] \tag{3.1}$$
$$\frac{d\psi_y}{d\eta} = -\varepsilon S \frac{u}{v}(\psi_x - \psi_y), \qquad \frac{d\psi_x(0)}{d\eta} = 0$$

where the terms containing derivatives of $\psi_x$ and $\psi_y$ with respect to coordinates are the greatest, and $x(\xi) = \psi_x(\eta)$ and $y(\xi) = \psi_y(\eta)$. As the boundary condition we specify that the derivative of $\psi_x(\eta)$ must vanish when $\eta = 0$. That condition yields the relation for the determination of one of the three constants appearing in the solution of system (3.1). The other two constants are determined by the method of joining the external to the internal solution. We represent the solution of (3.1) by the expansion in powers of the small parameter $\varepsilon$

$$\psi_{x,\,y}(\eta) = \sum_{m=0}^{\infty} \varepsilon^m \psi_{x,\,y}^{(m)}(\eta) \tag{3.2}$$

Solution of the system of zero approximation equations is

$$\psi_x^{(0)}(\eta) = h_{x0}^{(0)}, \qquad \psi_y^{(0)}(\eta) = h_{y0}^{(0)}$$

where $h_{x0}{}^{(0)}$ and $h_{y0}{}^{(0)}$ are constants determined by the method of joining. Functions $\psi_{x,\,y}^{(1)}$ satisfy the system of first approximation equations

$$\frac{d^2\psi_x^{(1)}}{d\eta^2} + \frac{d\psi_x^{(1)}}{d\eta} = -\,[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})]$$
$$\frac{d\psi_y^{(1)}}{d\eta} = -\frac{u}{v} S(h_{x0}^{(0)} - h_{y0}^{(0)}), \quad \frac{d\psi_x^{(1)}(0)}{d\eta} = 0$$

and are of the form

$$\psi_x^{(1)} = h_{x0}^{(1)} + h_x^{(1)}(\eta + e^{-\eta}), \qquad \psi_y^{(1)} = h_{y0}^{(1)} + h_{y1}^{(1)}\eta$$

The quantities $h_x{}^{(1)}$ and $h_{y1}{}^{(1)}$ are determined by the following functions of $h_{x0}{}^{(0)}$ and $h_{y0}{}^{(0)}$:

$$h_x^{(1)} = -\,[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})], \qquad h_{y1}^{(1)} = -\frac{u}{v} S(h_{x0}^{(0)} - h_{y0}^{(0)}) \tag{3.3}$$

and $h_{x0}{}^{(1)}$ and $h_{y0}{}^{(1)}$ are constants determined by joining the internal to the external expansion.

Functions of the second approximation expansion (3.2) satisfy the system of equations

$$\frac{d^2\psi_x^{(2)}}{d\eta^2} + \frac{d\psi_x^{(2)}}{d\eta} = -(2Kh_{x0}^{(0)} - S)\,\psi_x^{(1)}(\eta) - S(\psi_x^{(1)}(\eta) - \psi_y^{(1)}(\eta))$$
$$\frac{d\psi_y^{(2)}}{d\eta} = -\frac{u}{v} S(\psi_x^{(1)}(\eta) - \psi_y^{(1)}(\eta)), \qquad \frac{d\psi_x^{(2)}(0)}{d\eta} = 0$$

and are of the form

$$\psi_x^{(2)} = h_{x0}^{(2)} + h_{x1}^{(2)}(\eta + e^{-\eta}) + h_{x2}^{(2)}\frac{\eta^2}{2} + h_{x3}^{(2)}(\eta + 1)e^{-\eta}$$

$$\psi_y^{(2)} = h_{y0}^{(2)} + h_{y1}^{(2)}\eta + h_{y2}^{(2)}\frac{\eta^2}{2} + h_y^{(2)}e^{-\eta}$$

with $h_{x1}^{(2)}, h_{x2}^{(2)}, h_{x3}^{(2)}, h_{y1}^{(2)}, h_{y2}^{(2)}$ and $h_y^{(2)}$ are defined in terms of $h_{x0}^{(0)}, h_{y0}^{(0)}, h_{x0}^{(1)}$ and $h_{y0}^{(1)}$ by formulas

$$h_{x1}^{(2)} = -(2Kh_{x0}^{(0)} - S)\{[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})] + h_{x0}^{(1)}\} - \tag{3.4}$$
$$S\left\{\frac{u}{v}S(h_{x0}^{(0)} - h_{y0}^{(0)}) + h_{y0}^{(0)}\right\}, \quad h_{x2}^{(2)} = (2Kh_{x0}^{(0)} - S)[Kh_{x0}^{(0)2} -$$
$$S(h_{x0}^{(0)} - h_{y0}^{(0)})] + \frac{u}{v}S^2(h_{x0}^{(0)} - h_y^{(0)}), \quad h_{y1}^{(2)} = -\frac{u}{v}S(h_{x0}^{(1)} - h_{y0}^{(1)})$$
$$h_{y2}^{(2)} = \frac{u}{v}S\left\{[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})] - \frac{u}{v}S(h_{x0}^{(0)} - h_{y0}^{(0)})\right\}$$
$$h_y^{(2)} = -\frac{u}{v}S[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})]$$
$$h_{x3}^{(2)} = -(2Kh_{x0}^{(0)} - S)[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})]$$

**4.** We use the method of joining asymptotic expansions for determining the constants appearing in the external solution. We pass to the intermediate variables [7] $\eta = \eta^* / \mu(\varepsilon)$ and $\xi = 1(\varepsilon / \mu(\varepsilon))\eta^*$, assuming that $\varepsilon \ll \mu(\varepsilon) \ll 1$, write the external solution in the form of expansion in $\varepsilon / \mu(\varepsilon) \ll 1$

$$\chi_\gamma\left(1 - \frac{\varepsilon}{\mu(\varepsilon)}\eta^*\right) = \sum_{m=0}^{\infty}\varepsilon^m\left\{\chi_\gamma^{(m)}(1) - \frac{\varepsilon}{\mu(\varepsilon)}\eta^*\frac{d\chi_\gamma^{(m)}(1)}{d\xi} + \right. \tag{4.1}$$
$$\left.\left(\frac{\varepsilon}{\mu(\varepsilon)}\right)^2\frac{\eta^{*2}}{2}\frac{d^2\chi_\gamma^{(m)}(1)}{d\xi^2} + \ldots\right\}, \quad \gamma = x, y$$

It is convenient to express derivatives of functions $\chi_x^{(m)}$ and $\chi_y^{(m)}$ in terms of their values at the specified point by using equations of the appropriate approximations. For example, for $\xi = 1$ for the derivatives of $\chi_x^{(0)}, \chi_y^{(0)}, \chi_x^{(1)}$ and $\chi_y^{(1)}$ with respect to $\xi$ we have

$$\frac{d\chi_x^{(0)}}{d\xi} = K\chi_x^{(0)2} - S(\chi_x^{(0)} - \chi_y^{(0)}), \quad \frac{d\chi_y^{(0)}}{d\xi} = \frac{u}{v}S(\chi_x^{(0)} - \chi_y^{(0)}) \tag{4.2}$$
$$\frac{d^2\chi_x^{(0)}}{d\xi^2} = (2K\chi_x^{(0)} - S)[K\chi_x^{(0)2} - S(\chi_x^{(0)} - \chi_y^{(0)})] + \frac{u}{v}S^2(\chi_x^{(0)} - \chi_y^{(0)})$$
$$\frac{d^2\chi_y^{(0)}}{d\xi^2} = \frac{u}{v}S\left\{K\chi_x^{(0)2} - S\left(1 + \frac{u}{v}\right)(\chi_x^{(0)} - \chi_y^{(0)})\right\}$$
$$\frac{d\chi_x^{(1)}}{d\xi} = 2K\chi_x^{(0)}\chi_x^{(1)} - S(\chi_x^{(1)} - \chi_y^{(1)}) +$$
$$(2K\chi_x^{(0)} - S)[K\chi_x^{(0)2} - S(\chi_x^{(0)} - \chi_y^{(0)})] + \frac{u}{v}S^2(\chi_x^{(0)} - \chi_y^{(0)}),$$
$$\frac{d\chi_y^{(1)}}{d\xi} = \frac{u}{v}S(\chi_x^{(1)} - \chi_y^{(1)})$$

To join the internal to the external solution we equate the coefficients at like powers of the intermediate variable $\eta^*$ of the two solutions. In the zero approximation in terms of the small parameter $\varepsilon$ we have

$$\chi_x^{(0)}(1) = h_{x0}^{(0)}, \quad \frac{d\chi_x^{(0)}(1)}{d\xi} = [Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})] \tag{4.3}$$

$$\frac{d^2\chi_x^{(0)}(1)}{d\xi^2} = (2Kh_{x0}^{(0)} - S)[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})] + \frac{u}{v} S^2 (h_{x0}^{(0)} - h_{y0}^{(0)})$$

$$\chi_y^{(0)}(1) = h_{y0}^{(0)}, \quad \frac{d\chi_y^{(0)}(1)}{d\xi} = \frac{u}{v} S (h_{x0}^{(0)} - h_{y0}^{(0)})$$

$$\frac{d^2\chi_y^{(0)}(1)}{d^2\xi} = \frac{u}{v} S\left\{[Kh_{x0}^{(0)2} - S(h_{x0}^{(0)} - h_{y0}^{(0)})] - \frac{u}{v} S(h_{x0}^{(0)} - h_{y0}^{(0)})\right\}$$

In deriving this system the exponentially small terms were neglected in internal solutions. Equations which contain values of external solutions $\chi_x^{(0)}(1)$ and $\chi_y^{(0)}(1)$ determine constants $h_{x0}^{(0)} = \chi_x^{(0)}(1)$ and $h_{y0}^{(0)} = \chi_y^{(0)}(1)$. The substitution of these relationships into the remaining equations of system (4.3) and the use of derivatives at point $\xi = 1$, as defined in (4.2), makes it possible to ascertain directly that the second, third, fifth and sixth formulas, which correspond to the equality of coefficients at the first and second powers of the intermediate variable in the internal and external expansions, become identities.

Thus in the zero approximation in terms of the small parameter the internal solution is the same as the external within $(\varepsilon / \mu)^2$. Separating the part common to the external and internal solutions, we obtain in the first approximation in terms of $\varepsilon$ the solutions

$$x = \chi_x^{(0)}(\xi) + \varepsilon\chi_x^{(1)}(\xi) + \varepsilon h_x^{(1)} \exp\left\{-\frac{1-\xi}{\varepsilon}\right\}, \quad y = \chi_y^{(0)}(\xi) \tag{4.4}$$

which are equally valid on segment $0 \leqslant x \leqslant 1$. In the last equation $\chi_{x,y}^{(0)}(\xi)$ and $\chi_{x,y}^{(1)}(\xi)$ are external solutions of system (1.3). Solutions in the case of small coefficients of surface interaction were given in Sect. 1. Constant $h_x^{(1)}$ in (4.4) is determined by formula (3.3).

To remain within the accuracy specified for the zero approximation in terms of parameter $(\varepsilon / \mu) \ll 1$ it is sufficient to equate the coefficients at the zero and first powers of the intermediate variable $\eta^*$ in the external solution to the coefficients at the corresponding terms in the internal solution.

As the result we obtain

$$\chi_x^{(1)}(1) = h_{x0}^{(1)}, \quad d\chi_x^{(1)}(1)/d\xi = -h_{x1}^{(2)}$$
$$\chi_y^{(1)}(1) = h_{y0}^{(1)}, \quad d\chi_y^{(1)}(1)/d\xi = -h_{y1}^{(2)}$$

in which the first and third formulas determine $h_{x0}^{(1)} = \chi_x^{(1)}(1)$ and $h_{y0}^{(1)} = \chi_y^{(1)}(1)$. The substitution of these values into the second and fourth formulas with the use of (4.2) and (3.4) readily shows that they are identically satisfied.

Separating the common part with allowance for joining in the first approximation, we obtain the solution that is uniformly valid on the whole segment $0 \leqslant \xi \leqslant 1$

$$x(\xi) = \chi_x^{(0)}(\xi) + \varepsilon\chi_x^{(1)}(\xi) + \varepsilon^2\chi_x^{(2)}(\xi) + \varepsilon h_x^{(1)}(\eta + e^{-\eta}) + \varepsilon^2[h_x^{(2)}(\eta + e^{-\eta}) + h_{x3}^{(2)}e^{-\eta}(\eta + 1)] \tag{4.5}$$

$$y(\xi) = \chi_y^{(0)}(\xi) + \varepsilon\chi_y^{(1)}(\xi) + \varepsilon^2\chi_y^{(2)}(\xi) + \varepsilon^2 h_y^{(2)} e^{-\eta}$$

where $h_x^{(1)}$ and $h_x^{(2)}$, $h_{x3}^{(2)}$, $h_y^{(2)}$ are determined by (3.3) and (3.4).

The comparison of solutions (4.4) and (4.5) shows that the effect of the boundary layer on the reagent concentration in the bubble manifests itself in the first approximation.

In concluding we would point out that the method of solving the two-point boundary value problem developed here on the basis of joining asymptotic expansions makes it possible to calculate a reactor with a nonhomogeneous fluidized bed in one-dimensional approximation without having to resort to numerical computations.

Unlike the formal extension of the method [5] to the system of equations [10] the proposed method makes it possible to find a solution that is uniformly valid along the whole segment $0 \leqslant \xi \leqslant 1$ by first obtaining the analytic formula for related constants with the use of the device of joining the external and internal solutions.

The author thanks V. V. Struminskii for his interest in this paper.

REFERENCES

1. Aerov, M. E. and Todes, O. M., Hydraulic and Thermal Fundamentals of the Operation of Equipment with Steady and Boiling Granular Bed. Leningrad, "Khimiia", 1968.
2. Davidson, J. F. and Harrison, D., Fluidized Particles. Cambridge Univ. Press, New York, 1963.
3. Lewis, W. K., Gilliland, E. R. and Glass, W., Solid-catalized reaction in a fluidized bed. A. I. Ch. E. Journal, Vol. 5, № 4, 1959.
4. Slin'ko, M. G. and Sheplev, V. S., Simulation of catalytic processes in fluidized beds. Kinetika i Kataliz, Vol. 11, № 2, 1970.
5. Freeman, L. B. and Houchton, G., Singular perturbations of nonlinear boundary value problems arising in chemical flow reactors. Chem. Engrg. Sci., Vol. 21, № 11, 1966.
6. Burghardt, A. and Zaleski, T., Longitudinal dispersion at small and considerable Péclet numbers in chemical flow reactors. Chem. Engrg. Sci., Vol. 23, 1968.
7. Cole, J., Perturbation Methods in Applied Mathematics. Blaisdell Waltham, Mass., U. S. A., 1968.
8. Cohen, D. S., Multiple Selection of Nonlinear Partial Differential Equations. In: Lecture Notes in Mathematics, Springer-Verlag, Berlin, Heidelberg, New York, Vol. 322, 1973.
9. Chen, J. and O'Malley Jr., R. O., On the asymptotic solution of the two-parameter problem of the chemical reactor theory. SIAM J. Appl. Math., Vol. 26, № 4, 1974.
10. Rahman, M., Some aspects of perturbation solution arising in nonisothermal tubular chemical flow reactors. Chem. Engrg. Sci., Vol. 29, № 11, 1974.

Translated by J. J. D.